Friction, No Friction,… Rolling or Sliding of an object?

In order to clarify confusion between rolling and sliding, two cases are developed in this paper:

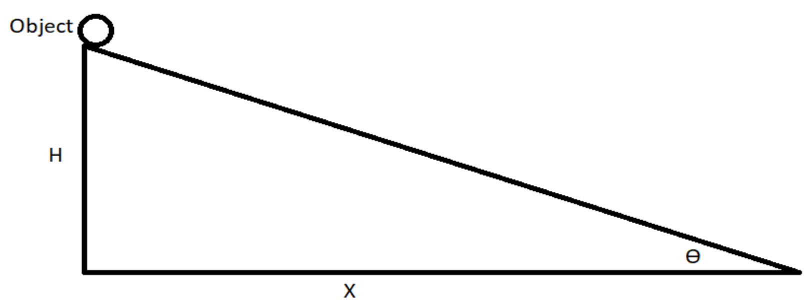
1. Rolling happens only if the surface is rough, that's why it is said "Friction causes rolling" so no rolling on a frictionless surface.

2. Sliding happens on a frictionless surface and, or if friction is present but the object is not round so cannot role.

Question 1: Over a ramp:

1. 4 different objects; a hollow thin cylinder, a solid cylinder, a hollow thin sphere and one solid sphere all with mass "M" and radius "R" are released one at a time, from the same point at the top of a rough ramp which makes angle ϴ with horizontal. At what speed each one hits the end line?

2. Object is at the top of the same ramp but no friction, compare the answers.



Solution:

1. Use Rotational Kinematic in order to find the velocity, by applying "the Law of Conservation of Energy":

$$\Delta P_E = \Delta K_E \text{ (Linear + Rotational)}$$

$$MgH = \tfrac{1}{2} MV^2 + \tfrac{1}{2} I\omega^2 \quad (1)$$

"I" is the momentum of inertia and ω is the angular velocity of the rolling object.

a. Hollow thin cylinder:

$$I = MR^2, \qquad \text{and } V = \omega R \quad \text{or} \quad \omega = V/R$$

(1) $MgH = \frac{1}{2}MV^2 + \frac{1}{2}MR^2(V/R)^2 = MV^2$ (J)

Therefore: $V = \sqrt{gH}$ (m/s)

b. Solid cylinder:

$I = \frac{1}{2}MR^2$

(1) $MgH = \frac{1}{2}MV^2 + \frac{1}{4}MR^2(V/R)^2 = \frac{3}{4}MV^2$ (J)

Therefore: $V = 2\sqrt{\frac{gH}{3}} = 1.155\sqrt{gH}$ (m/s)

c. Hollow sphere:

$I = \frac{2}{3}MR^2$

(1) $MgH = \frac{1}{2}MV^2 + \frac{1}{2} \cdot \frac{2}{3}MR^2(V/R)^2 = \frac{5}{6}MV^2$ (J)

Therefore: $V = \sqrt{\frac{6gH}{5}} = 1.095\sqrt{gH}$ (m/s)

d. Solid sphere:

$I = \frac{2}{5}MR^2$

$MgH = \frac{1}{2}MV^2 + \frac{1}{2} \cdot \frac{2}{5}MR^2(V/R)^2 = \frac{7}{10}MV^2$ (J)

Therefore: $V = \sqrt{\frac{10gH}{7}} = 1.195\sqrt{gH}$ (m/s)

Conclusion: The order of fast to slow is; solid sphere, solid cylinder, hollow sphere and hollow cylinder. The reason is about the contribution of their momentum of inertia in total energy. Larger "I" means larger rotational kinetic energy, so smaller linear kinetic energy therefore less velocity.

2. In frictionless surface the object will slide but not role and the shape of the object is not important. The final speed calculated as follow:

$$MgH = 1/2MV^2 \text{ therefore:}$$

$$V = \sqrt{2gH} = 1.414\sqrt{gH} \text{ (m/s)}$$

Which is the largest speed.

Question 2: The surface is spherical:

1. A ball is rolling down from the top, over a rough sphere with a radius of R without slipping, at what angle the ball will leave the surface of the sphere?

Assume 2 possibilities, solid and hallo sphere.

Solution:

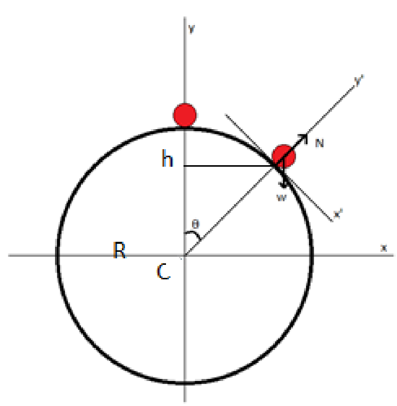
Hint: In this case the object is rolling, because the friction exist, means the friction force causes rolling.

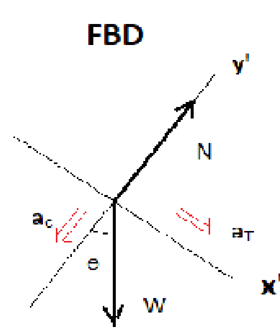
Let's take the coordinate system (x' y') with the object in order to ease the calculation of Radial or

Centripetal ($a_y = a_c$) acceleration and Tangential ($a_T = a_x$) acceleration.

Note: The Friction can be added to FBD on the positive x direction because it causes the rolling and tangential acceleration a_T and it is equal to W.sinΘ, but it will not contribute to calculation of speed if we apply the law of conservation of energy

Apply Newton's law in Y direction according the FBD:





$$N - W.\cos\Theta = N - mg.\cos\Theta = - ma_c$$

The object will leave the surface if N=0, therefore:

$$a_c = g.\cos\Theta = v^2/R \qquad (1)$$

As we can see a_c is a function of Θ, calculus must be applied to Rotational Kinematic in order to find the velocity, but it is easier to apply the Law of Conservation of Energy: $P_E = K_E$ (Linear + Rotational)

$$mgh = ½\, mv^2 + ½\, I\omega^2$$

"I" is the momentum of inertia and ω is the angular velocity of the rolling object with radius r.

Case 1, Solid sphere:

$$I = \frac{2}{5}mr^2 \quad \text{and} \quad \omega = 2\pi r \qquad \text{then use } v = \omega r$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mr^2\,(v/r)^2 = \frac{7}{10}mv^2$$

$$\text{Therefore: } v^2 = \frac{10}{7}gh$$

Use Trigonometry: $h = R – R.\cos\Theta = R(1- \cos\Theta) \qquad (2)$

$$v^2 = \frac{10}{7}gh = \frac{10}{7}g\,R(1- \cos\Theta)$$

Substituting v^2 in (1): $g.\cos\Theta = \frac{10}{7}g\,(1- \cos\Theta)$

Then: $\frac{10}{7}\cos\Theta + \cos\Theta = \frac{10}{7}$ $\qquad \frac{17}{7}\cos\Theta = \frac{10}{7}$

Therefor: $\Theta = \cos^{-1}(\frac{10}{17}) = 53.97 = 54$ Degrees if the ball is solid.

Case 2, Hollow sphere:

$$I = \frac{2}{3}mr^2 \quad \text{and} \quad \omega = 2\pi r \qquad \text{then use } v = \omega r$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{3}mr^2\,(v/r)^2 = \frac{5}{6}mv^2$$

$$\text{Therefore: } v^2 = \frac{6}{5}gh$$

Use (2) for h we find:

$$v^2 = \frac{6}{5}g\,R(1- \cos\Theta)$$

Substituting v^2 in (1): $g.\cos\Theta = \frac{6}{5}g\,(1- \cos\Theta)$

Then: $\frac{6}{5}\cos\Theta + \cos\Theta = \frac{6}{5}$ $\frac{11}{5}\cos\Theta = \frac{6}{5}$

Therefor: $\Theta = \cos^{-1}(\frac{6}{11}) = 56.94 = 57$ Degrees if the ball is hollow.

Note: In that case the ball is assumed to be small and its radius "r" has been ignored compare to the sphere radius. For more accurate calculation the rolling radius should be considered as "R+r"

2. An object is sliding down from the top, over a frictionless sphere with a radius of R, at what angle it will separate from the sphere surface?

Solution:

We use the same diagrams and coordinate system (x' y') as previous question with Centripetal ($a_y = a_c$) and tangential ($a_T = a_x$) accelerations:

Apply Newton's law in Y direction according the FBD:

$$Y: \quad N - W.\cos\Theta = N - mg.\cos\Theta = -ma_c$$

In which N is the Normal Force and W is mg, the force of gravity

The object will leave the surface if N = 0, therefore:

$$a_c = g.\cos\Theta = v^2/R \qquad (1)$$

The net force is variable during the motion, calculus must be applied to kinematic in order to find the velocity, and so it will be easier if we use the Law of Conservation of Energy to eliminate V:

$$\Delta P_E = \Delta K_E$$

$$mgh = \tfrac{1}{2} mv^2 \qquad \text{then: } v^2 = 2gh$$

To eliminate h substitute it by (2)

$$v^2 = 2g\,R(1 - \cos\Theta)$$

Substituting v^2 in (1):

$$g.\cos\Theta = 2g\,(1 - \cos\Theta)$$

So: $3\cos\Theta = 2$,

Then: $\Theta = \cos^{-1}(2/3) = 48.18$ Degrees

Which is smallest angle compare to all previous cases because it moves faster.